### UNIT – 03

#### Planes

## Unit-03/Lecture-01

**Introduction**[**RGPV**/ **June-2002,04,06,11 Dec.2001,02,03,06,10,11**]- In this chapter, we deal with two dimensional objects called planes. Planes have length, breadth and negligible thickness. Only those planes are considered in the chapter whose shape can be defined geometrically and are regular in nature.

### ORIENTATIONS OF PLANES

The possible orientations of the surface of a plane with respect to the principal planes are given below-

- 1. Surface of plane is parallel to the H.P. (and perpendicular to the V.P.).
- 2. Surface of plane is parallel to the V.P. (and perpendicular to the H.P.).
- 3. Surface of plane is perpendicular to both H.P. and V.P. (i.e., parallel to profile plane).
- 4. Surface of plane is inclined to the H.P. and perpendicular to the V.P.
- 5. Surface of plane is inclined to the V.P. and perpendicular to the H.P.
- 6. Surface of plane is inclined to both H.P. and V.P.

### Initially, we will consider the first five orientations given above, in the first angle.

### **PROJECTIONS OF PLANES**

In this topic various plane figures are the objects.

What is usually asked in the problem...?

To draw their projections means F.V., T.V. & S.V.

What will be given in the problem?

1. Description of the plane figure.

2. Its position with HP and VP.

### In which manner its position with HP & VP will be described?

1. Inclination of its SURFACE with one of the reference planes will be given.

2. Inclination of one of its EDGES with other reference plane will be given

(Hence this will be a case of an object inclined to both reference Planes.)

### TRACES OF PLANES

A plane which is not parallel to any of the principal planes will meet the reference planes in a line, extended if necessary. This line is called trace of the plane.

Horizontal trace (H.T.) when the plane meets the H.P. in the line, extended if necessary, then that line called horizontal trace.



And If surface is assumed // to VP – its FV will show True Shape.

3. Hence begin withdrawing TV or FV as True Shape.

4. While drawing this True Shape – Keep one side/edge (which is making inclination) perpendicular to xy line.

**Problem no.1:-** Rectangle 30mm and 50mm sides is resting on HP on one small side which is 30<sup>°</sup> inclined to VP, while the surface of the plane makes 45<sup>°</sup> inclination with HP. Draw its projections.

R.G.T.U. June 2002

Solution:-Read problem and answer following questions

- 1. Surface inclined to which plane? ------ HP
- 2. Assumption for initial position? -----// to HP
- 3. So which view will show True shape? --- TV
- 4. Which side will be vertical? ---One small side.

### Hence begin with TV, draw rectangle below X-Y, Drawing one small side vertical.



**Problem no.2:** A  $30^{\circ} - 60^{\circ}$  set square of longest side 100 mm long is in VP and  $30^{\circ}$  inclined to HP while its surface is  $45^{\circ}$  inclined to VP. Draw its projections.

R.G.T.U. June/July 2006

### Solution:-

Read problem and answer following questions

- 1 .Surface inclined to which plane? ------ VP
- 2. Assumption for initial position? -----// to VP
- 3. So which view will show True shape? --- FV
- 4. Which side will be vertical? -----longest side.



**Problem no.3:-** A regular pentagon of 30 mm sides is resting on HP on one of its sides with its surface  $45^{\circ}$  inclined to HP. Draw its projections when the side in HP makes  $30^{\circ}$  angle with VP.

R.G.T.U. Dec 2006

Solution:-

### Read problem and answer following questions

- 1. Surface inclined to which plane? ------ HP
- 2. Assumption for initial position? ----- // to HP
- 3. So which view will show True shape? --- TV
- 4. Which side will be vertical? ------ Any side.

Hence begin with TV, draw pentagon below X-Y line, taking one side vertical.



**Problem no.5:**-A hexagonal lamina has its one side in HP and Its apposite parallel side is 25mm above HP and In VP. Draw its projections. Take side of hexagon 30 mm long. Solution:-

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# Read problem and answer following questions

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- 1. Surface inclined to which plane? ------ HP
- 2. Assumption for initial position? ----- // to HP
- 3. So which view will show True shape? --- TV
- 4. Which diameter horizontal? ------ AC

Hence begin with TV, draw rhombus below X-Y line, taking longer diagonal // to X-Y.

# ONLY CHANGE is the manner in which surface inclination is described :-

One side on Hp & it's opposite side 25 mm above Hp. Hence redraw 1st Fv as a 2nd Fv making above arrangement. Keep a'b' on xy & d'e' 25 mm above xy.



S.NO	RGPV QUESTIONS	Year	Marks
Q.1	A hexagonal lamina of 24mm side has its surface	Dec 2014	7
	inclined at 30° to H.P. Its one side is lying on the H.P.		
	and inclined at 45° to V.P. Draw its projections.		
Q.2	A regular pentagonal lamina ABCDE of edge length		
	50mm is resting on its side AB in V.P. so that the	Dec 2011	16
	lamina is inclined at $30^{\circ}$ to V.P. Line joining vertex D		
	to the midpoint of opposite side, is inclined $30^{\circ}$ to		
	H.P. Draw the projections of the lamina.		
Q.3	A pentagon of 30mm side has one corner on H.P. Its	June 2010	8
	plane is inclined at $60^{\circ}$ to V.P. and perpendicular to		
	H.P. Draw the projection of the pentagon.		
Q.4	raw the projections of a circle of 50mm diameter	June 2003	16
	resting in the HP on a point A on the circumference,		
	its plane is inclined at 45 <sup>°</sup> to the HP and-		
	(a) The top view of the diameter AB making $30^{\circ}$		
	angle with the VP		
	(b) The diameter AB making 30 <sup>0</sup> angle with the VP		
Q.4	A hexagonal plate of 30mm side is resting on one of	Dec 2010	10
	its corner on H.P. The plate is perpendicular to V.P.		
	and inclined to H.P. at 45 <sup>0</sup> . Draw its projections.		

### FREELY SUSPENDED CASES

**Problem no.6:-** An isosceles triangle of 40 mm long base side, 60 mm long altitude Is freely suspended from one corner of Base side. Its plane is 45<sup>0</sup> inclined to VP. Draw its projections. **R.G.T.U. Jan/Feb 2006, June 2003,01** 

### **IMPORTANT POINTS:-**

1. In this case the plane of the figure always remains *perpendicular to Hp.* 

2. It may remain parallel or inclined to VP.

3. Hence *TV* in this case will be always a *LINE view*.

4. Assuming surface // to VP, draw true shape in suspended position as FV.

(Here keep line joining point of contact & centroid of fig. vertical )

5. Always begin with FV as a True Shape but in a suspended position. AS shown in 1st FV.



**Problem no.7:-** A semicircle of 100 mm diameter is suspended from a point on its straight edge 30 mm from the midpoint of that edge so that the surface makes an angle of 45<sup>°</sup> with VP. Draw its projections.

### **IMPORTANT POINTS:-**

1. In this case the plane of the figure always remains *perpendicular to Hp.* 

2. It may remain parallel or inclined to VP.

3. Hence *TV* in this case will be always a *LINE view*.

4. Assuming surface // to VP, draw true shape in suspended position as FV. (Here keep line joining **point of contact & centroid of fig. vertical** )

5. Always begin with FV as a True Shape but in a suspended position. AS shown in 1st FV.



### To determine true shape of plane figure when its projections are given. BY USING AUXILIARY PLANE METHOD

### Follow the below given steps:-

1. Draw the given Fv & TV as per the given information in problem.

2. Then among all lines of Fv & TV select a line showing True Length (T.L.) (Its other view must be // to xy)

3. Draw  $x_1$ - $y_1$  perpendicular to this line showing T.L.

4. Project view on  $x_1$ - $y_1$  (it must be a line view)

5. Draw x2-y2 // to this line view & project new view on it.

It will be the required answer i.e. True Shape.

NOW FINAL VIEWS ARE ALWAYS SOME SHAPE, NOT LINE VIEWS:

SO APPLYING ABOVE METHOD:

WE FIRST CONVERT ONE VIEW IN INCLINED LINE VIEW .(By using  $x_1y_1$  auxiliary plane) THEN BY MAKING IT // TO  $X_2$ - $Y_2$  WE GET TRUE SHAPE.

**Problem no.8:-** TV is a triangle abc, ab is 50 mm long, angle cab is  $30^{\circ}$  and angle cba is  $65^{\circ}$ . a'b'c' is a FV. a' is 25 mm, b' is 40 mm and c' is 10 mm above HP respectively. Draw projections of that figure and find its true shape.

### As per the procedure-

- 1. First draw FV & TV as per the data.
- 2. In TV line ab is // to xy hence its other view a'b' is TL. So draw  $x_1y_1$  perpendicular to it.
- 3. Project view on  $x_1y_1$ .



a) First draw projectors from a', b' & c' on  $x_1y_1$ .

b) From xy take distances of a, b & c (TV) mark on these projectors from  $x_1y_1$ . Name points  $a_1b_1$  &

#### c<sub>1</sub>.

c) This line view is an Aux. TV. Draw x<sub>2</sub>y<sub>2</sub> // to this line view and project Aux. FV on it. for that from x<sub>1</sub>y<sub>1</sub> take distances of a', b' & c' and mark from x<sub>2</sub>y= on new projectors.
4. Name points a'<sub>1</sub> b'<sub>1</sub> & c'<sub>1</sub> and join them. This will be the required true shape.

**Problem no.9:-** Draw a regular pentagon of 30 mm sides with one side 30<sup>°</sup> inclined to xy. This figure is TV of some plane whose FV is A line 45<sup>°</sup> inclined to xy. Determine its true shape.

Solution:-

In this case also true length is not available in any view, but actually we do not require TL to find its true shape, as one view (FV) is already a line view. So just by drawing  $x_1y_1$  // to this view we can project view on it and get true shape.



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Q.1 A 4 a p	A hexagonal prism, side of base 20mm and axis 48mm long, rests with its base on H.P. such that an edge of the base is parallel to V.P. Draw the projections of the prism on an auxiliary plane which makes an angle of 60 <sup>0</sup> with the H.P.	June 2010	8



**Icosahedrons-** It is a solid, having twenty equal equilateral triangular faces.

**Prism-** This is a polyhedron, having two equal and similar regular polygons called its ends or bases, parallel to each other and joined by other faces which are rectangles. The imaginary line joining the centre of the bases is called the axis.

A right and regular prism has its axis perpendicular to the base. All the faces are equal rectangles.

**Pyramid-** This is a polyhedron, having a regular polygon as a base and a number of triangular faces meeting at a point called the vertex or apex. The imaginary line joining the apex with the centre of the base is known as the axis.

A right and regular pyramid has its axis perpendicular to the base which is a regular plane. All the faces are isosceles triangles.

**Solid of revolution-** These solids are obtained by revolving a plane figure like a rectangle, triangle or a semi-circle about an edge which remains fixed.

**Cylinder**- A cylinder is obtained by revolving a rectangle about a fixed line called an axis. **Cone**- A cone is obtained by revolving a right-angled triangle about its perpendicular side which remains fixed.

**Sphere-** A sphere is obtained by revolving a semi-circle about its diameter which remains fixed. As a result of revolution, a spherical surface with its diameter equal to the diameter of the semi-circle is generated.

Dimensional parameters of different solids-







**Problem no.1:-** A square pyramid, 40 mm base sides and axis 60 mm long, has a triangular face on the ground and the vertical plane containing the axis makes an angle of 45<sup>°</sup> with the VP. Draw its projections. Take apex nearer to VP.

### Solution Steps :-

Triangular face on Hp, means it is lying on HP:-

1. Assume it standing on HP.

2. Its TV will show True Shape of base( square)

3. Draw square of 40mm sides with one side vertical TV & taking 50 mm axis project FV. ( a triangle)

4. Name all points as shown in illustration.

5. Draw  $2^{nd}$  FV in lying position I.e. o'c'd' face on xy, And project its TV.

6. Make visible lines dark and hidden dotted, as per the procedure.

7. Then construct remaining inclination with VP

(VP containing axis ic the center line of  $2^{nd}$  TV. Make it  $45^0$  to xy as shown take apex near to xy, as it is nearer to VP) & project final FV.



#### For dark and dotted lines-

- 1. Draw proper outline of new view DARK.
- 2. Decide direction of an observer.
- 3. Select nearest point to observer and draw all lines starting from it-dark.
- 4. Select farthest point to observer and draw all lines (remaining) from it- dotted.

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	A hexagonal pyramid of base side 30mm and axis 60mm, has an edge of its base on the ground inclined at 45 degree to the V.P. and the axis is inclined at 30 degree to the H.P. Draw its projections.	Dec 2014	7
Q.2	A pentagonal pyramid, side of base 25mm and height 60mm has one of its slant faces on the Horizontal plane and the edge of the base contained by that slant face makes an angle of 30° to the V.P. Draw its projections.	June 2013	7
Q.3	A pentagonal pyramid with 25mm side base and 65mm height has one of its slant faces on the horizontal plane and the edge of the base contained by that slant face makes an angle of $25^{\circ}$ to the V.P. Draw the projections of the pyramid.	June 2013	7

**Problem no.2:-** A cone 40 mm diameter and 50 mm axis is resting on one generator on HP which makes  $30^{\circ}$  inclination with VP Draw its projections.

[R.G.T.U. Dec 2011]

### Solution Steps :-

Resting on Hp on one generator, means lying on HP:

- 1. Assume it standing on HP.
- 2. Its TV will show True Shape of base( circle )
- 3. Draw 40mm dia. Circle as TV & taking 50 mm axis project FV. ( a triangle)
- 4. Name all points as shown in illustration.
- 5. Draw 2<sup>nd</sup> FV in lying position I.e. o'e' on xy. And project its TV below xy.
- 6. Make visible lines dark and hidden dotted, as per the procedure.

7. Then construct remaining inclination with VP (generator  $o_1e_1 30^0$  to xy as shown) & project final FV.

#### For dark and dotted lines-

- 1. Draw proper outline of new vie DARK.
- 2. Decide direction of an observer.
- 3. Select nearest point to observer and draw all lines starting from it-dark.
- 4. Select farthest point to observer and draw all lines (remaining) from it- dotted.



**Problem no.3:-** A cylinder 40 mm diameter and 50 mm axis is resting on one point of a base circle on VP while its axis makes  $45^{\circ}$  with VP and FV of the axis  $35^{\circ}$  with HP. Draw projections.

R.G.T.U. April 2009

#### **Solution Steps :-**

Resting on VP on one point of base, means inclined to VP:-

1. Assume it standing on VP

2. Its FV will show True Shape of base & top (circle)

- 3. Draw 40mm dia. Circle as FV & taking 50 mm axis project TV. ( a Rectangle)
- 4. Name all points as shown in illustration.
- 5. Draw  $2^{nd}$  TV making axis  $45^0$  to xy And project its FV above xy.

6. Make visible lines dark and hidden dotted, as per the procedure.

7. Then construct remaining inclination with HP (FV of axis I.e. centre line of view to xy as shown) & project final TV.



S.NO	RGPV QUESTIONS	Year	Marks
Q.1	A right circular cone, diameter of base 50mm and height 65mm, lies on one of its generators on H.P. Such that the generator is inclined to V.P. at 30 <sup>0</sup> . Draw its projections.	Dec 2011	7
Q.2	A right circular cone of dia. 70mm and axis height 80mm is resting on one of its generators in H.P. The top view of the axis is inclined at 45 <sup>°</sup> to V.P. Draw the projections of the cone.	Dec 2010	10
Q.3			

**Problem no.4:** A square pyramid 30 mm base side and 50 mm long axis is resting on its apex on HP, such that its one slant edge is vertical and a triangular face through it is perpendicular to VP. Draw its projections.

### R.G.T.U. Dec 2001

### Solution Steps :-

1. Assume it standing on HP but as said on apex.( inverted )

2. Its TV will show True Shape of base( square)

3. Draw a corner case square of 30 mm sides as TV (as shown) Showing all slant edges dotted, as those will not be visible from top.

4. Taking 50 mm axis project FV. ( a triangle)

5. Name all points as shown in illustration.

6. Draw 2<sup>nd</sup> FV keeping o'a' slant edge vertical & project its TV

7. Make visible lines dark and hidden dotted, as per the procedure.

8. Then redrew  $2^{nd}$  TV as final TV keeping  $a_1o_1d_1$  triangular face perpendicular to VP I.e. xy. Then as usual project final FV.



**Problem no.5:-** A cube of 50 mm long edges is so placed on Hp on one corner that a body diagonal is parallel to Hp and perpendicular to VP Draw its projections.

### Solution Steps :-

1. Assuming standing on HP, begin with TV, a square with all sides equally inclined to xy. Project FV and name all points of FV & TV.

2. Draw a body-diagonal joining c' with 3'( This can become // to xy)

3. From 1' drop a perpendicular on this and name it p'

4. Draw 2<sup>nd</sup> FV in which 1'-p' line is vertical *means* c'-3' diagonal must be horizontal. Now as

usual project TV.

6. In final TV draw same diagonal is perpendicular to VP as said in problem. Then as usual project final FV.



S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Draw the projections of a cone, base 50 mm diameter and axis 60mm long, resting on H.P. on a point of its base circle with the axis making an angle of 30 degree with H.P. and its top view making an angle of 45 degree withV.P.	Dec 2014	7
Q.2	A square pyramid of base 50mm side and 80mm high is held in such a way that one of its edges connecting one of the corners of the base and the apex is perpendicular to the ground and parallels to VP Draw the projections.	Dec 2001	16
Q.3	A hexagonal pyramid with side of base 30mm and axis 75mm long has an edge of the base on V.P. and inclined at $30^{\circ}$ to H.P. The triangular face containing the edge makes an angle of $45^{\circ}$ with V.P. Draw the projections of the solid.	June 2012	16

**Problem no.6:-** A tetrahedron of 50 mm long edges is resting on one edge on HP while one triangular face containing this edge is vertical and 45<sup>°</sup> inclined to VP. Draw projections.

R.G.T.U. Jan/Feb 2006

### Solution Steps :-

As it is resting assume it standing on HP.

Begin with TV , an equilateral triangle as side case as shown:-

First project base points of Fv on xy, name those & axis line.

From a' with TL of edge, 50 mm, cut on axis line & mark o' (as axis is not known, o' is finalized by slant edge length) Then complete FV.

In 2<sup>nd</sup> FV make face o'b'c' vertical as said in problem And like all previous problems solve completely.



**IMPORTANT:** - Tetrahedron is a special type of triangular pyramid in which base sides & slant edges are equal in length. Solid of four faces. Like cube it is also described by One dimension only. Axis length generally not given.



**Problem no.8:-** A cube of 50 mm long edges is so placed on HP on one corner that a body diagonal through this corner is perpendicular to HP and parallel to VP Draw its three views.

#### Solution Steps :-

1. Assuming it standing on HP begin with TV, a square of corner case.

2. Project corresponding FV & name all points as usual in both views.

3. Join a'1' as body diagonal and draw 2<sup>nd</sup> FV making it vertical (I' on xy)

4. Project its TV drawing dark and dotted lines as per the procedure.

5. With standard method construct Left-hand side view.

(Draw a 45<sup>0</sup> inclined Line in TV region (below xy). Project horizontally all points of TV on this line and reflect vertically upward, above xy. After this, draw horizontal lines, from all points of FV, to meet these lines. Name points of intersections and join properly. For dark & dotted lines locate observer on left side of Fv as shown.)



S.NO	RGPV QUESTIONS	Year	Marks
Q.1	7. A triangular prism of base 30mm side and axis 50mm long is resting on H.P. on one of its bases with a face perpendicular to V.P. Draw the projections of the solid.	June 2012	8
Q.2	4. A pentagonal pyramid, side of base 25mm and length of axis 50mm lies on one of its slant edges with its axis parallel to V.P. Draw the projections of the pyramid.	June 2012	7

**Problem no.9:-** A right circular cone, 40 mm base diameter and 60 mm long axis is resting on HP on one point of base circle such that its axis makes 45<sup>°</sup> inclination with HP and 40<sup>°</sup> inclination with VP. Draw its projections.

#### **Solution Steps :-**

In previous all cases 2<sup>nd</sup> inclination was done by a parameter not showing TL. Like TV of axis is inclined to VP etc. But here it is clearly said that the axis is 40<sup>0</sup> inclined to VP. Means here TL inclination is expected. So the same construction done in those Problems is done here also. See carefully the final TV and inclination taken there. **So assuming it standing on HP begin as usual.** 



**Problem no.10:-** A triangular prism, 40 mm base side 60 mm axis is lying on HP on one rectangular face with axis perpendicular to VP. One square pyramid is leaning on its face centrally with axis // to VP. Its base side is 30 mm & axis is 60 mm long resting on HP on one edge of base. Draw FV & TV of both solids. Project another FV on an AVP 45<sup>o</sup> inclined to VP.

### Solution Steps :-

Draw Fv of lying prism (an equilateral Triangle) And Fv of a leaning pyramid. Project TV of both solids.

Draw  $x_1y_1 45^0$  inclined to xy and project aux. FV on it.

Mark the distances of first FV from first xy for the distances of aux. FV from  $x_1y_1$  line. Note the observer's directions Shown by arrows and further steps carefully.



**Problem no.11:-** A hexagonal prism of base side 30 mm long and axis 40 mm long, is standing on HP on its base with one base edge // to VP. A tetrahedron is placed centrally on the top of it. The base of tetrahedron is a triangle formed by joining alternate corners of top of prism. Draw projections of both solids. Project an auxiliary TV on AIP 45<sup>0</sup> inclined to HP. **R.G.T.U. June 2010** 

#### **Solution Steps :-**

Draw a regular hexagon as TV of standing prism With one side // to xy and name the top points. Project its FV – a rectangle and name its top.

Now join its alternate corners a-c-e and the triangle formed is base of a tetrahedron as said. Locate center of this triangle & locate apex o Extending its axis line upward mark apex o' By cutting TL of edge of tetrahedron equal to a-c and complete FV of tetrahedron.

Draw an AIP  $(x_1y_1) 45^0$  inclined to xy And project Aux. TV on it by using similar Steps like previous problem.



REFERENCCE			
воок	AUTHOR	PRIORITY	
Engineering Graphics	N. D. Bhatt	1	
Engineering Graphics	C M Agrawal	2	